Elliptic Flow Analysis at RHIC: Fluctuations vs. Non-Flow Effects

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The cumulant method is applied to study elliptic flow (v_2) in Au+Au collisions at $\sqrt{s} = 200$AGeV, with the UrQMD model. In this approach, the true event plane is known and both the non-flow effects and event-by-event spatial (ϵ) and v_2 fluctuations exist. Qualitatively, the hierarchy of v_2's from two, four and six-particle cumulants is consistent with the STAR data, however, the magnitude of v_2 in the UrQMD model is only 60% of the data. We find that the four and six-particle cumulants are good measures of the real elliptic flow over a wide range of centralities except for the most central and very peripheral events. There the cumulant method is affected by the v_2 fluctuations. In mid-central collisions, the four and six-particle cumulants are shown to give a good estimation of the true differential v_2, especially at large transverse momentum, where the two-particle cumulant method is heavily affected by the non-flow effects.

To create extremely hot and dense matter with partons as its fundamental components - called the Quark-Gluon Plasma (QGP) - is a major goal of current and future high energy heavy-ion collisions experiments at SPS, RHIC and LHC [1]. However, due to the complex nature of the relativistic nucleus-nucleus reactions, the QGP, if it has been created, escapes direct detection. Therefore, in order to distinguish the existence and later on to investigate the properties of the new state of matter, one must find observables which allow to deduce the properties of the intermediate (QGP) state from the final state hadrons. Elliptic flow (v_2), which is the second Fourier harmonic [2] in the transverse distribution of the emitted particles, is expected to be sensitive to the early pressure gradients and therefore the equation of state (EOS) of the formed fire-ball in the heavy-ion collisions [2, 3]. Recent elliptic flow results on Au+Au collisions at $\sqrt{s} = 200$ AGeV [4, 5, 6, 7] indeed indicate high pressure gradients in the early stage of the reaction and might therefore hint towards the existence of an intermediate QGP state at this energy.

In principle, the elliptic flow of hadrons at low transverse momenta (p_T) can be related to the degree of thermalization, the viscosity and the EOS of the produced matter [3, 8, 9]. On the other hand, the elliptic flow of the high p_T particles is related to jet fragmentation and energy loss of the primordially produced hard Anti-quark-Quark pair when traveling through the hot QCD medium [10]. At low p_T, results from most of the RHIC experiments [4, 5, 6, 7] indicate a gradual increase of v_2 with the increase of p_T. This behavior is approximately consistent with the prediction of relativistic hydrodynamical calculation with a first order phase transition to a QGP [9]. When $p_T \geq 1.5$ GeV/c, the v_2 begins to saturate and eventually decreases [4, 11]. This is a clear signal for the breakdown of hydrodynamics at intermediate p_T and a transition towards jet physics. As shown in [10], the v_2 at large p_T might be a sensitive probe of the initial parton density distribution of the Quark-Gluon matter produced. Thus an accurate v_2 measurement might allow deeper insights into the bulk properties of the produced matter.

However, an unambiguous experimental measurement of the elliptic flow is not a trivial task due to the unknown orientation of the reaction plane. Often, experiments use the so called reaction plane method [12] to extract the magnitude of the elliptic flow. In this method, the reaction plane is fixed according to the flow vector of the event, then the estimated v_2 with respect to the chosen reaction plane is corrected for the event plane resolution, which accounts for the error in the deduction of the reaction plane. The original reaction plane method is consistent with the two-particle correlation method [4, 12, 13], in which v_2 is related to the two particle angular difference $\phi_1 - \phi_2$ by $v_2 = \sqrt{\langle cos2(\phi_1 - \phi_2)\rangle}$. However, these two-particle correlations based methods might suffer from effects which are not related to the reaction plane, these additional contributions are usually called non-flow effects [14], such as the overall transverse momentum conservation, small angle azimuthal correlations due to final state interactions, resonance decays, jet production [15] and quantum correlations due to the HBT effect [16]. In order to eliminate the non-flow contributions to the measured collective flow in the reaction plane method, a rapidity gap between the particles used to estimate the reaction plane and the measured particles is usually introduced. But whether this improvement works well is still not clear. Recently, the cumulant method was proposed [17] to diminish the non-flow effects. The idea of the cumulant method is to extract flow with many-particle cumulants, which are the many-particle correlations with subtraction of the contributions from the correlations due to the lower-order multiplets. It is believed that the pure many-particle non-flow corre-

lations have much less contributions to the measured flow in the many-particle cumulant method. In other words, the many-particle cumulant method should be much less sensitive to non-flow effects [17]. At RHIC energy, the cumulant method has been applied by STAR [4, 13, 18] and PHENIX [5] in the flow analysis of Au+Au collisions. It is found that the integral v_2 from the two-particle correlations which is denoted as $v_2\{2\}$ is about 15% larger than the values from four and six-particle cumulants($v_2\{4\}$ and $v_2\{6\}$). One might attribute the extra v_2 in $v_2\{2\}$ to the non-flow correlations and conclude that the non-flow contribution has been successfully eliminated in the results with four or six-particle cumulants [4]. However, as indicated in [13, 19], the v_2 from many-particle cumulants is also affected by the event-by-event v_2 fluctuations. For a rough estimation of the fluctuations' contribution to the measured elliptic flow, the reader is referred to [19]. There, the estimation is based on the assumption that the v_2 of an event is proportional to initial eccentricity of the nucleons or quarks [20]. The authors found that the difference between $v_2\{2\}$ and $v_2\{4\}$ can also be explained by a definite amount of the fluctuations of v_2 which gives a larger $v_2\{2\}$ and a smaller $v_2\{4\}$ than the exact v_2 for the semi-central collisions. Here it is worth stressing that this conclusion can be extended to the cumulant analysis of any order harmonics v_n with $n \geq 1$, i.e. the v_n fluctuations result in $|v_n\{2\}| > |v_n\{4\}|$ generally. Therefore, both non-flow correlations and v_2 fluctuations can explain the fact that $v_2\{2\} > v_2\{4\}$ at RHIC. If the non-flow effects are dominant, the exact v_2 is $v_2\{4\}$ or $v_2\{6\}$; if the fluctuations are dominant, the exact v_2 is $(v_2\{2\} + v_2\{4\})/2$ according to [19]. This ambiguity poses large systematic error to the v_2 measurement especially at large p_t, where the difference between $v_2\{2\}$ and $v_2\{4\}$ is most prominent. So it is essential to make clear which effect is dominant in the difference between $v_2\{2\}$ and $v_2\{4\}$ in order to reduce the large systematic uncertainty.

In this article, we use the UrQMD model (v2.2) [21, 22] to test the robustness of the cumulant method for the elliptic flow analysis. The advantages of using a transport approach compared to hydrodynamics are immanent:

- Firstly, transport models do not make any additional assumptions on local/global equilibration of the matter created during the collisions, but treat the non-equilibrium processes directly.

- Secondly, the present transport approach includes most non-flow correlations such as the overall transverse momentum conservation, small angle azimuthal correlations due to final state interactions, resonance decay and jet prodcution, naturally during the systems evolution.

- Thirdly, the UrQMD model is an event by event model, hence it contains the event by event fluctuations of the elliptic flow.

Finally, the reaction plane angle Φ_R is known in the model, which allows the direct calculation of the exact elliptic flow from its basic definition, that is $v_2 = \langle \cos 2(\phi - \Phi_R) \rangle$. Therefore, the UrQMD model, even if for the time being still underpredicts the integral v_2 in $\sqrt{s}$=200AGeV Au-Au collisions at RHIC, is an ideal tool to find out whether the v_2 fluctuations and non-flow effects have large effects on the experimentally used cumulant method.

Before the application of the cumulant method, let us begin by examining the magnitude of the fluctuations of eccentricity and v_2. Fig.1 shows a scatter plot of initial spatial eccentricity ($\epsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$) of the participants as a function of impact parameter based on a subset of the available UrQMD minimum-bias events. As one can see, the eccentricity fluctuations in the model are in magnitude similar to the eccentricity itself. Note that the magnitude of the fluctuations is quite similar to the estimates calculated with a Monte Carlo Glauber model [19]. Due to the large event by event fluctuations, $\langle \epsilon^2 \rangle^{1/2}$, $\langle \epsilon^4 \rangle^{1/4}$ and $\langle \epsilon^6 \rangle^{1/6}$ are much larger than $\langle \epsilon \rangle$, especially for the most central and peripheral events. The eccentricity fluctuations are supposed to be the main origin of the v_2 fluctuations. Fig.2 is the scatter plot of the event v_2 averaged over all particles with $|\eta| < 2.5$, as a function of impact parameter based on the same subset of UrQMD minimum-bias events. Like the eccentricity, the event v_2 fluctuations are also of the same magnitude as the v_2 itself. Therefore, $\langle v_2^2 \rangle^{1/2}$, $\langle v_2^4 \rangle^{1/4}$ and $\langle v_2^6 \rangle^{1/6}$ are also much larger than $\langle v_2 \rangle$, especially in the most central and very peripheral centralities where the $\langle v_2 \rangle$ is very small. Fig.3 shows the correlation of v_2 and eccentricity in the semi-central collisions. The fact that the $\langle v_2 \rangle$ is proportional to eccentricity, shows that the fluctuations in the initial conditions are the major origin of v_2 fluctuations [20] besides the statistical noise [23] due to the limited number of particles used to estimate v_2.

The observation of these large fluctuations puts some doubt on the accuracy of the experimental methods for the extraction of the elliptic flow parameters. Therefore, we will now focus on the cumulant method and compare the model results (with fluctuations and non-flow effects) obtained by different order cumulant methods with the exact v_2. For the detailed application of the cumulant method, the reader is referred to [17]. In our analysis we use unit weights in the evaluation of the generating function of the cumulants. The parameter r_0 is 1.5 as usually used in previous analysis. Actually, the detailed investigation indicates that the present results are rather insensitive to the r_0 values as pointed out before in [17]. For the integral v_2 analysis, we use all particles in the pseudorapidity region $|\eta| < 2.5$, and the number of particles from event to event fluctuates in each centrality bin. We have tested the cumulant method with fixed number of particles in each event for the same centrality and find that the results do not change within the present statistical error. The centralities in our analysis are selected according to the same geometrical fractions
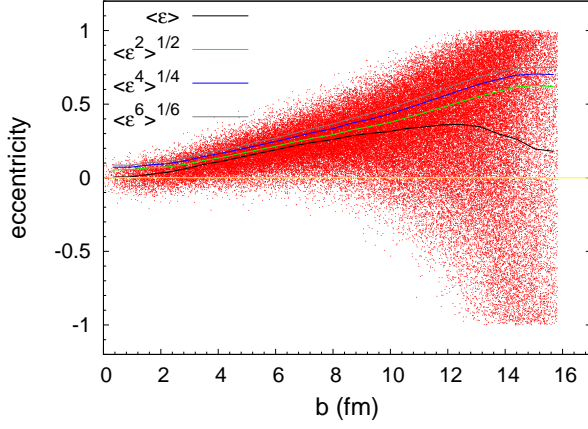
FIG. 1: (Color online) Scatter plot of the initial spatial eccentricities ($\epsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$) of the participants at different impact parameters from the UrQMD model. The black, green, blue and grey lines are the average eccentricity $\langle \epsilon \rangle$, $\langle \epsilon^2 \rangle^{1/2}$, $\langle \epsilon^4 \rangle^{1/4}$ and $\langle \epsilon^6 \rangle^{1/6}$ respectively.
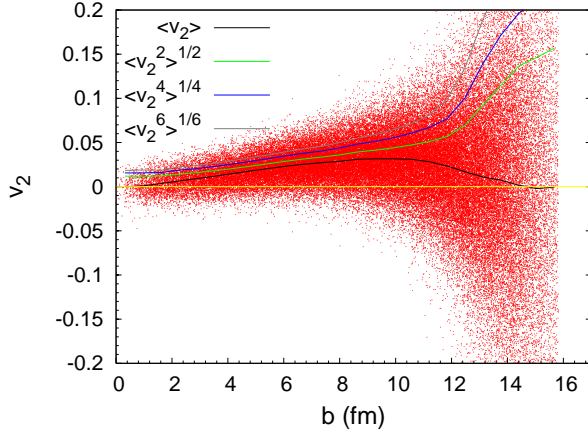

FIG. 2: (Color online) Scatter plot of the event v_2 averaged over all particles within $|\eta| < 2.5$ at different impact parameters from the UrQMD model. The black, green, blue and grey lines are the average elliptic flow $\langle v_2 \rangle$, $\langle v_2^2 \rangle^{1/2}$, $\langle v_2^4 \rangle^{1/4}$ and $\langle v_2^6 \rangle^{1/6}$ respectively.

of the total cross section (0-5%,5-10%,10-20%,20-30%,30-40%,40-50%,50-60%,60-70%) as used by the STAR experiment [4], however, we use impact parameter cuts instead of multiplicity cuts. More than $1.3 \cdot 10^6$ minimum bias events are used in the integral v_2 analysis. In order to increase the statistics at the most peripheral centrality bin(60-70%), additional $7 \cdot 10^5$ events are added in this centrality bin.

Fig.4 shows the calculated integral v_2 results as a function of centrality. The elliptic flow parameters extracted from two-particle cumulant $v_2\{2\}$ deviate rather strongly from the theoretically expected v_2 as obtained from the known reaction plane. In fact, at all centralities, $v_2\{2\}$
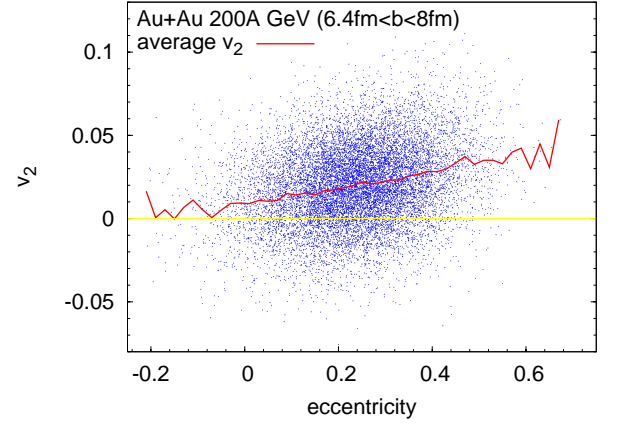

FIG. 3: (Color online) Scatter plot of the event v_2 averaged over all particles within $|\eta| < 2.5$ at different eccentricities from the UrQMD model. The red line is the average elliptic flow.

is larger than the exact v_2 by 18%. However, for mid-central collisions ($\sigma/\sigma_{\text{tot}} \sim 10 - 50\%$), the elliptic flow from four particle ($v_2\{4\}$) and six particle cumulants ($v_2\{6\}$) show almost no difference and both agree well with the exact v_2. This fact is quite contrary to the prediction in [19], that is, the exact v_2 should be in the middle of the $v_2\{2\}$ and $v_2\{4\}$ if the differences between the cumulant methods are mainly due to v_2 fluctuations. Therefore, we conclude that for semi-central to semi-peripheral centralities the contribution of the v_2 fluctuations to the cumulant results is almost negligible and the difference between $v_2\{2\}$ and $v_2\{4\}$ or $v_2\{6\}$, is mainly due to non-flow effects in the UrQMD model. The similar conclusion is also made in [24], where the directed flow v_1 in 1.69A GeV Ru+Ru collisions at SIS/GSI is measured with the cumulant method. It is found that $|v_1\{2\}|$ is less than $|v_1\{4\}|$ which is also contrary to the predictions based on flow fluctuations.

In spite of the good agreement of $v_2\{4\}$ or $v_2\{6\}$ with the exact v_2 in semi-central bins, from Fig.4, we have also seen that both $v_2\{4\}$ and $v_2\{6\}$ do not agree with the exact v_2 in the most central and the very peripheral bins. This means at central and very peripheral collisions, the v_2 fluctuations indeed play an important role as indicated in [19]. In the peripheral bins the higher order cumulants give larger v_2 than the exact one. In the most central bin, the $v_2\{4\}$ is smaller and even becomes complex (not shown in Fig.4) due to the fluctuations, while the $v_2\{6\}$ is slightly larger than the exact v_2. These findings are qualitatively consistent with previous results within a simplified Monte-Carlo Glauber treatment [19].

In order to estimate how sensitive the cumulant method is to impact parameter fluctuations in a centrality bin, we also performed the cumulant analysis in enlarged centrality bins. The pink (grey) points in Fig.4 show the results for the enlarged bins(0-10%, 5-20%, 10-30%, 20-40%, 30-50%, 40-60%, 50-70%). One can see

that the v_2 values from any order cumulants are still in line with the corresponding v_2 results from the original bins although the impact parameter fluctuations in the enlarged bins are larger than those in the original (narrower) centrality bins. Thus, the main contribution to the v_2 fluctuations in the original centrality bins should be due to v_2 fluctuations at the same impact parameter, e.g. due to the spatial eccentricity fluctuations from event to event and not due to impact parameter fluctuations.

While the total elliptic flow values extracted from the calculation are lower than the experimental results, the relations between $v_2\{2\}$, $v_2\{4\}$ and $v_2\{6\}$ are similar to the results reported by the STAR collaboration at RHIC. As shown in Fig.5(A), open symbols denote the calculation, while full symbols show the STAR data on the ratios $v_2\{2\}/v_2\{4\}$ and $v_2\{6\}/v_2\{4\}$ for comparison. The good agreement between UrQMD results and the data may indicate that the mechanism which accounts for the differences between $v_2\{2\}$ and $v_2\{4\}$ or $v_2\{6\}$ is the same. In Fig.5(B), we show the g_2 factor from the UrQMD model. The g_2 factor, is defined in experiments as [25] $g_2 = N \cdot (v_2\{2\}^2 - v_2\{4\}^2)$, where N is the event multiplicity (for our analysis) or the number of wounded nucleons (for the STAR data) which should be approximately proportional to the multiplicity. The g_2 is a measure of the non-flow effects and should be independent of the centrality as originally suggested by [25]. However, the STAR [4] and SPS [26] data show that with the increase of the impact parameter, the g_2 will decrease by about a factor of 3. This decrease of the observed g_2 is consistent with the results based on the eccentricity (or v_2) fluctuations [19], which seems confirm the conjecture in [25], i.e. the v_2 fluctuations account for the variance of g_2 with centrality. As we can see in Fig.5(B), the g_2 from the UrQMD model (the blank square) also has similar shape as the data (please note that g_2 from UrQMD has been rescaled by a factor 0.186 to compare to the 200AGeV STAR data, since the magnitude of the v_2 is too small and N in the g_2 defintion is also different). However, the decrease of g_2 in the UrQMD model is not mainly from v_2 fluctuations. Since the exact v_2 is known in the UrQMD model, the exact two-particle non-flow correlations (or the exact g_2), i.e. $N \cdot (v_2\{2\}^2 - v_2^2)$, is also known. In Fig.5(B), the solid line is the exact g_2 from the UrQMD model. Apparently, the exact g_2 decreases towards peripheral centralities too. Therefore, although the v_2 fluctuations indeed make the experimentally measured g_2 decrease slightly faster, the non-flow correlations themselves change with the centralities too.

Recently, to overcome the experimental limitations in the v_2 measurement with the reaction plane method, the STAR experiment has upgraded its set-up. The Shower Max detector of the Zero Degree Calorimeters(ZDC-SMD) has been added to reconstruct the reaction plane with the sideward deflection (bounce-off) of the spectator neutrons. The non-flow effects are supposed to be minimal, because the spectator neutrons barely participate

in the complicated final state rescattering. The STAR preliminary results [27] for the measured v_2 with respect to this reaction plane is denoted as $v_2\{ZDC\text{-}SMD\}$. The reported $v_2\{ZDC\text{-}SMD\}$ agrees well with $v_2\{4\}$ in the mid-central collisions(10-50%). $v_2\{ZDC\text{-}SMD\}$ is larger than $v_2\{4\}$ in the most central bins(0-5% and 5-10%) and smaller than the $v_2\{4\}$ in the very peripheral bins(larger than 50%). The relation between $v_2\{ZDC\text{-}SMD\}$ and $v_2\{4\}$ is similar with those between exact v_2 and $v_2\{4\}$ from UrQMD. This similarity, on the one hand, confirms that the mechanism which affects the cumulant method is indeed the same as that in UrQMD; on the other hand, it supports that the flow measurement with the ZDC-SMD is not disturbed by non-flow effects or flow fluctuations. Therefore we want to advocate the ZDC-SMD method for the flow analysis, because it allows to extract very reliable flow results over the whole centrality.
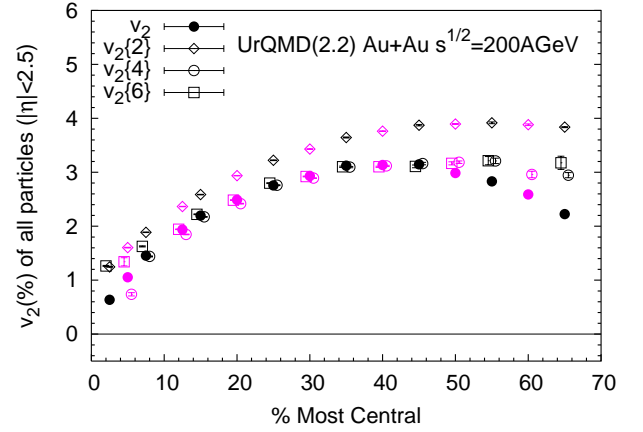

FIG. 4: (Color online) The integral v_2 results($v_2\{2\}$,$v_2\{4\}$ and $v_2\{6\}$) from the cumulant method are compared to the exact v_2 in different centrality bins. The pink (grey) points are the corresponding results from the enlarged centrality bins which merge two of the original bins.

Let us now turn to the study of the the differential v_2. In the cumulant method, the differential v_2 in one p_T or rapidity bin is estimated with the cumulants between the particles in this bin and those in one common "pool". The average v_2 of the particles in the "pool" should be known from the integral flow analysis. For the following differential v_2 analysis, we always use all the particles within $|\eta| < 2.5$ as the "pool". One should also notice that the non-flow correlations which affect the differential flow analysis will be that between the particles in the chosen bin and those in the "pool".

Firstly, let us explore the p_T dependence of v_2. Here we use more than $6 \cdot 10^5$ semi-central events (with impact parameters from 6.7 to 8.3 fm corresponding to about 20% to 30% of the total cross section). From the above results on the integral v_2, we know that both four and six-particle cumulants produce almost the exact v_2 in this centrality bin, but it is still necessary to see whether the different cumulant method produce the differential
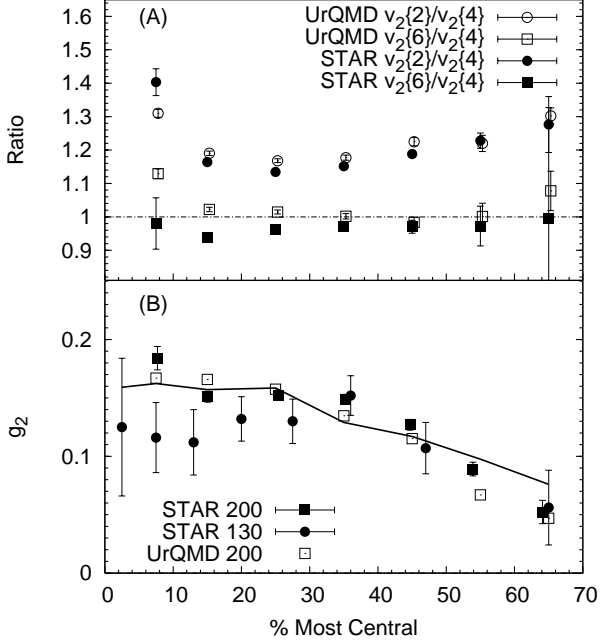
FIG. 5: (A) The ratios $v_2\{2\}/v_2\{4\}$ and $v_2\{6\}/v_2\{4\}$ from UrQMD are compared to the STAR data [4]. (B) The g_2 factors from the UrQMD model are compared to the STAR data, the solid line is the exact g_2, i.e. $N\cdot(v_2\{2\}^2-v_2^2)$ (please see the text for details). Note that g_2's from UrQMD have been scaled down by a factor 0.186

FIG. 6: $v_2(p_T)$ in the semi-central collisions: results from the cumulant method are compared to the exact v_2

v_2 correctly. Especially at large transverse momenta (p_T) non-flow contributions are expected to be large and might influence the results obtained by the cumulant method. Fig.6 shows the calculations for the v_2 of particles within $|\eta| < 2.5$ as a function of p_T. At low p_T, the exact v_2 increases with the increase of p_T and reaches a maximum at about 2.5 GeV/c, then drops down with a further increase of p_T. In contrast, the v_2 from two-particle cumulant $v_2\{2\}$ increases also at low p_T, but stays roughly constant at large p_T, in addition it is always higher than the exact v_2. The saturation of $v_2\{2\}$ is consistent with STAR's $v_2\{2\}$ results [11]. This strong deviations point towards substantial contributions from non-flow effects in the two-particle cumulant method. The higher order cumulants do a much better job in reproducing the exact v_2. Here, the difference between $v_2\{4\}$ and the exact v_2 is much smaller especially at large p_T. However, $v_2\{4\}$ is still larger than the exact v_2, indicating that even four-particle cumulants are not free from non-flow disturbances. When we go to the six-particle cumulant results $v_2\{6\}$, we get good agreement with the exact v_2 in the whole p_T range within the statistical error. This shows that the non-flow effects have been completely eliminated in $v_2\{6\}$.

Finally, we will study the pseudo-rapidity (η) dependence of v_2 with the cumulant method using the same set
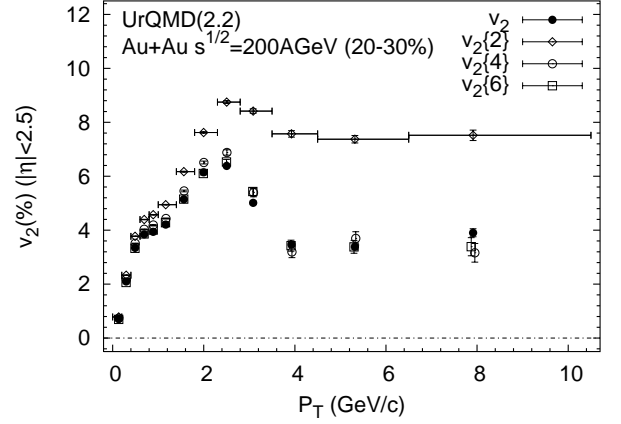
of semi-central events as for transverse momentum analysis. It is usually expected that at large η, the non-flow effects are less important than at mid-rapidity because of the larger rapidity gap between the particles in the rapidity bin and the "pool" particles. So the difference between $v_2\{2\}$ and $v_2\{4\}$ might be smaller at large η compared to midrapidity. Fig.7(A) shows the results on $v_2(\eta)$ obtained from the different methods. Indeed one observes that at large η, $v_2\{2\}$, $v_2\{4\}$ and $v_2\{6\}$ are almost similar and they all agree well with the exact v_2. This is in line with the STAR results on the $v_2(\eta)$ also indicating agreement between $v_2\{2\}$ and $v_2\{4\}$ at large η [4]. However, the smaller difference between the v_2's from any-order cumulants at larger rapidity must not be taken as a sign that the non-flow effects are less important at larger rapidities, because the v_2 itself decreases towards large rapidity. To demonstrate this, Fig.7(B) shows the ratios of $v_2\{n\}$ over the exact v_2. One observes that the ratios are roughly independent of the rapidity. Therefore, the non-flow effects at forward rapidity might be as important as those at mid-rapidity.

After the test of the cumulant method, we know that v_2 from the two-particle cumulant is heavily affected by the non-flow effects. While the still remaining question is what is the main origin of these non-flow correlations. Firstly, in the UrQMD model, there is no correlations due to the HBT effect. The correlations due to the overall transverse momentum conservation are also found to be less important for the v_2 measurement [14]. The short range direct correlations due to resonance decay, jet production and the last elastic or inelastic interaction before freeze-out can be removed by selecting the particles in the analysis preferentially. That is to say, only one of the final state particles that are **directly** from the above processes is selected in the analysis. The integral $v_2\{2\}$ and $v_2\{4\}$ of the selected particles in the semi-central bin (20-30%) are almost the same as those of the original analysis. Therefore, the short range direct correlations are not the main non-flow correlations at least for this
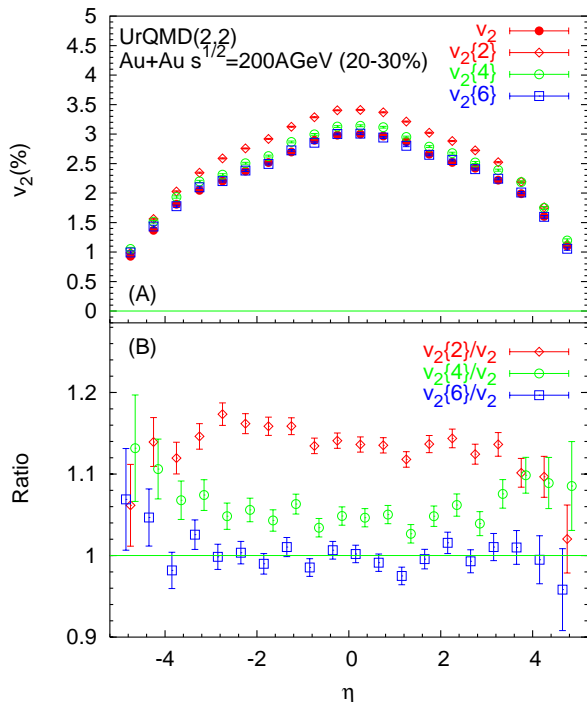
FIG. 7: (Color online) (A) $v_2(\eta)$ in semi-central Au+Au collisions at $\sqrt{s} = 200$ AGeV. Results from the cumulant method are compared to the exact v_2. (B) Ratios of $v_2\{n\}$ over the exact v_2.

centrality. It should be stressed that the jet processes happen at the very beginning of the event and most resonances decay before the freeze-out of the system. Therefore, most of the daughter particles directly from jets or resonances will interact with other particles in the system. And the correlations between these daughter particles could be changed by the final state interactions. As has been shown in [28], the final state rescatterings, or more exactly, the transverse radial expansion can contribute to the non-flow correlations between these daughter particles due to the fact that the particles from the same NN collsions (or jets and resonances) are pushed in the same direction radially in the transverse plane. Ref. [28] predicts that, this kind of non-flow correlations will depend on the centrality (following the development of radial flow). It is indeed observed both in the UrQMD results and in the data that g_2 factor increases towards central collisions, which shows the non-flow correlations between the daughter particles of jets and resonances due to transverse radial expansion could be very important or even dominant for semi-central to central collisions.

In summary, we have applied the cumulant method to analyze the v_2 of the Au+Au reactions at $\sqrt{s} = 200$ AGeV within the UrQMD model. On the integral v_2 analysis, we reproduce the hierarchy of $v_2\{2\}$, $v_2\{4\}$ and $v_2\{6\}$ observed by the STAR experiment even if the v_2 from UrQMD is only about 60% of the data. From the comparisons of the cumulant results to the exact v_2,

we found that v_2 fluctuations affect the results from the cumulant method in the most central and very peripheral collisions. However, this effect is almost negligible over a wide range of the mid-central collisions (about 10-50% of the total cross section).

While the two-particle cumulant results are heavily affected by non-flow effects, non-flow effects can indeed be nearly eliminated using four and six-particle cumulants. The similarity between STAR data and UrQMD results shows that the new flow measurements at STAR (using the ZDC-SMD detector) are a good way to obtain v_2 values which are not disturbed by the non-flow effects and v_2 fluctuations over the whole centrality range.

For the differential v_2 analysis, the two-particle cumulant method gives a nearly saturated v_2 at large p_T in stark contrast to the exact v_2 that drops down rapidly at high p_T. The v_2 from four and six-particle cumulants agree well with the exact v_2 especially at large p_T. However, there are still some non-flow contributions left in the four-particle cumulant method so that the $v_2\{4\}$ is always a little (about 4%) larger than the exact v_2. Finally, we point out that in the present model the non-flow effects at forward rapidity might be as important as those at mid-rapidity.

As for the origin of the non-flow correlations, we find that the short-range direct correlations due to resonance decay, jet production and the last final state interactions before freeze-out are not important for the semi-central to central Au-Au collsions at RHIC. While the indirect correlations between the daughter particles of jets and resonances could be the major non-flow effects for these collisions, especially these correlations could be substantionally enhanced by the transverse radial expansion of the medium [28].

A final remark. As shown above, the many-particle cumulant method $v_2\{n \geq 4\}$ allows for a good estimation of the exact v_2 in the mid-central collisions, thus one may justify other analysis methods by comparing their results with the cumulant method results. For instance, the PHENIX reaction plane method [5] seems also to suffer from non-flow effects because it gives the same $v_2(p_T)$ results as the two-particle cumulant method which is heavily affected by the non-flow effects as discussed above. The STAR ZDC-SMD method seems to give good estimates of the integral flow. But further comparisons with the cumulant method on the differential flow are necessary to fully justify its application in the differential flow analysis.

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